Plume-rise modelling: sensitivity analysis and umbrella cloud formation
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Overview

• Part I:
  • estimating the total mass released by Eyjafjallajökull in 2010
  • sensitivity to changes in the entrainment coefficients

• Part II:
  • a simple plume model for umbrella cloud formation
Part 1. Estimating total mass emitted by Eyjafjallajökull in 2010 using plume-rise models
Estimating the total mass emitted by Eyjafjallajökull in 2010 using a plume-rise model

- Estimate mass flux over duration of eruption (14 April - 23 May)
- Integrate mass flux to get total emitted mass
  - compare with observations (Gudmundsson et al. 2012)
  - compare with equivalent result using Mastin’s formula (Mastin et al. 2009)
- Meteorological profiles updated every three hours
  – taken from Met Office’s NWP model (Unified Model)
- Time series of eruption height (Webster et al. 2012)
Estimating the total mass emitted by Eyjafjallajökull in 2010 using a plume-rise model

Green: plume-rise model ($z_{max}$ only)

Blue: plume-rise model ($z_{max} +$ radius)

Red: Mastin’s formula

Black: observed value

Results sensitive to entrainment coefficients
Bent-over plumes

\[ z_{\text{max}} \]

(a) \( \tilde{U} \ll 1 \)

(b) \( \tilde{U} \gg 1 \)

\[ b_{\text{max}} \to \infty \quad \text{as} \quad U \to 0 \]
Bent-over plumes

Red: $z_{\text{max}} + b_{\text{max}}$

Blue: $z_{\text{max}}$

$\tilde{U} = \frac{U}{F_0^{1/4} N^{1/4}}$

$z_{\text{top}} = \min(z_{\text{max}}|U=0, z_{\text{max}}+b_{\text{max}})$

Mastin (JGR, 2014)
Sensitivity to entrainment constants

- Entrainment

\[ E = 2\pi b \rho_a (|\alpha \Delta u_s|)^m + (|\beta \Delta u_n|)^m)^{1/m} \]

- Bent-over plume:
  - height: \( z_{\text{max}} \propto \beta^{-2/3} Q_0^{1/3} U^{-1/3} N^{-2/3} \)
  - mass flux: \( Q_0 \propto \beta^2 z_{\text{max}}^3 N^2 U \)

- For bent-over plume \( b = \beta z \):

\[ z_{\text{top}} = z_{\text{max}} + b_{\text{max}} \propto \left( \frac{1}{\beta^{2/3}} + \beta^{1/3} \right) \frac{Q_0^{1/3}}{N^{2/3} U^{1/3}} \]

- Mass flux:

\[ Q_0 \propto \frac{\beta^2}{(1 + \beta)^3} z_{\text{top}}^3 N^2 U \]
Sensitivity to entrainment constants

(a) Theoretical

\[ f(\beta) = \frac{\beta^2}{(1 + \beta)^3}, \quad g(\beta) = \beta^2 \]

(b) Realistic model

NB: \( \beta = 1 \) is upper bound on physically plausible values of \( \beta \)
Part 2. Simple plume model for umbrella cloud formation
Model for co-flowing region

- Extend model of Bloomfield and Kerr (JFM, 2000) to volcanic case
- Two sets of equations: upward and downward
  - equations for fluxes of mass, vertical momentum, enthalpy and moisture
  - \[ u_\alpha = \alpha \left( \frac{\rho_u}{\rho_d} \right)^{1/2} \bar{w}_u, \quad u_\beta = \beta \left( \frac{\rho_d}{\rho_u} \right)^{1/2} \bar{w}_d, \quad u_\gamma = \gamma \left( \frac{\rho_d}{\rho_a} \right)^{1/2} \bar{w}_d \]
- Buoyancy is defined relative to ambient
- Solve upward and downward equations iteratively
Results: fixed MER, no wind

**BK original:** $\alpha = 0.085$, $\beta = 0.147$, $\gamma = 0.147$

**DRT (JFM 2010):** $\alpha = 0.05$, $\beta = 0.8$, $\gamma = 0.01$
Conclusions

• Estimate of total mass released by Eyjafjallajökull in 2010
  • plume-rise model agrees well with observations if radius taken into account (also true of Mastin’s formula)
  • result is sensitive to changes in entrainment coefficients
  • sensitivity to changes in $\beta$ reduced when radius taken into account

• Umbrella cloud model
  • interacting upward and downward plumes
  • slight change in maximum rise height
  • upward mass flux decreases towards top of plume
  • provide estimate of mass flux for initialising intrusion model
  • lots of scope for systematic comparison with 3D models